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Inelastic final-state interactions in $B \to VV \to \pi K$ processes

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Abstract. We study the final-state interactions in $B \to \pi K$ decays through $B \to VV \to \pi K$ processes where the inelastic rescattering occurs via single pion exchange. The next-to-leading order low-energy effective Hamiltonian and Bauer-Stech-Wirbel (BSW) model are used to evaluate the weak transition matrix elements and final-state interactions. We found that the final-state interaction effects in $B \to \rho K^* \to \pi K$ processes are significant. The Fleischer-Mannel relation for the Cabibbo-Kobayashi-Maskawa (CKM) angle γ can be significantly modified.

PACS. 13.25.Hw Decays of bottom mesons – 13.75.Lb Meson-meson interaction – 11.30.Er Charge conjugation, parity, time reversal, and other discrete symmetries – 13.40.Hq Electromagnetic decays

1 Introduction

Final-state interactions (FSI) play an important role in many physical processes, especially in the weak decays which are in the focus of recent interests. Their effects lie in two aspects. First, strong phases in the weak decay amplitudes are generated by the final-state interactions, they can contribute the strong phases for the direct CP asymmetries. Second, FSI effects can significantly change theoretical predictions for certain quantities. The study of final-state interactions would definitely need an information about the non-perturbative effects of low-energy hadron interactions. Unless we can correctly evaluate the FSI effects, it is impossible to extract reliable information about the reaction mechanism or new physics from the data. Not only the understanding of the final-state interaction effects in weak decays is crucially important, but it is also a challenging and difficult task in both theory and phenomenology. Up to now, definite quantitative analysis has not been accessible yet. The estimate of the final-state interactions is centred on some particular cases where the symmetry relations can be applied, or one can use some simplified models, for example, the Regge pole or single-pion exchange model, etc. to do the job.

The origin of CP violation in the Standard Model comes from the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In general, for three generation quark families the CKM matrix elements form a unitarity triangle. So, the reasonable extraction of each angle (α, β, γ) is extremely important for testing the Standard Model. Among the three angles, to extract γ is most difficult. Some methods [1] have been put forward for this purpose. But all the methods are either too complicated or impractical from the experimental point of view. Last year, the CLEO collaboration reported the combined ratios for $B \to \pi K$ decays [2]:

$$\begin{aligned} BR(B^{\pm} \to \pi^{\pm} K) &= (2.3^{+1.1+0.2}_{-1.0-0.2} \pm 0.2) \times 10^{-5}\,, \\ BR(B_d \to \pi^{\mp} K^{\pm}) &= (1.5^{+0.5+0.1}_{-0.4-0.1} \pm 0.1) \times 10^{-5}\,. \end{aligned}$$

Fleischer and Mannel [3] gave a boundary relation for the CKM angle γ based on the above results: $R \geq \sin^2\gamma$, where $R = \frac{BR(B_d \to \pi^{\mp} K^{\pm})}{BR(B^{\pm} \to \pi^{\pm})K} = 0.65 \pm 0.40$, $\gamma \equiv Arg(V_{ub}^*)$. In their work, final-state interactions were neglected. However, the final-state interactions in such channels may be important and cannot be ignored. Some authors [4,5] have studied the final-state interaction effects in $B \to \pi K$ decays. Investigations of [4] are based on the Regge pole theory. In [6], the authors study final-state interactions due to single pion exchange in $D \to VP$ processes. Their results show that the single pion exchange can be significant. It is believed that even though the two schemes describe the process based on different physical considerations, in practice, each of them works well, just because the uncertainties are partly compensated by proper selection of some phenomenological parameters in these schemes.

In this paper, we study the final-state interactions in $B \to VV \to \pi K$ due to single pion exchange. In Standard Model calculations, the decay amplitude of the process $B \to \rho K^*$ is of the same order as that of $B \to \pi K$.

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In fact, both $K^* \to K\pi$ and $\rho \to \pi\pi$ are the dominant strong decay channels, so the single pion exchange mechanism may dominate the final state interactions. In our processes, the exchanged pion is in the t-channel. There are other two-vector-meson states which can rescatter into the πK final state by pion exchange. But these intermediate mesons have smaller couplings to πK and $\pi\pi$ than $K^* \to \pi K$ and $\rho \to \pi\pi$; they are also heavier than ρ and K^* and will result in larger t-values, that will reduce their contribution even more. So, among the final-state interaction processes $B \to VV \to \pi K$, $B \to \rho K^* \to \pi K$ should be the largest one.

We use the method which was presented in [6]. The next-to-leading order low-energy effective Hamiltonian and Bauer-Stech-Wirbel (BSW) model [7] are used to evaluate the weak transition matrix elements and final-state interactions. We find that the final-state interaction effects in $B \to \rho K^* \to \pi K$ are significant.

2 Formulation of FSI in the effective Hamiltonian

To evaluate the decays $B \to \pi K$, we use the next-to-leading order low-energy effective Hamiltonian and BSW model. The next-to-leading order low-energy effective Hamiltonian describing $|\Delta B| = 1$ transitions is given at the renormalization scale $\mu = O(m_b)$ [8] as follows:

$$\mathcal{H}_{eff}(|\Delta B| = 1) = \frac{G_F}{\sqrt{2}}\Bigg[\sum_{q=u,c} v_q \Bigg\{ Q_1^q C_1(\mu) + Q_2^q C_2(\mu) + \sum_{k=3}^{10} Q_k C_k(\mu)\Bigg\}\Bigg] + H.C. \quad (1)$$

The CKM factors v_q are defined as $v_q = V_{qs}^* V_{qb}$, where $q = u,\ c$.

Ten operators $Q_1^u,\ Q_2^u,\ Q_3, \ldots, Q_{10}$ are known as the following forms:

$$\begin{aligned}
Q_1^u &= (\bar{q}_\alpha u_\beta)_{V-A}(\bar{u}_\beta b_\alpha)_{V-A}\\
Q_2^u &= (\bar{q}u)_{V-A}(\bar{u}b)_{V-A}\\
Q_{3(5)} &= (\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V-A(V+A)}\\
Q_{4(6)} &= (\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}\\
Q_{7(9)} &= \tfrac{3}{2}(\bar{q}b)_{V-A}\sum_{q'} e_{q'}(\bar{q}'q')_{V+A(V-A)}\\
Q_{8(10)} &= \tfrac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A(V-A)}\,,
\end{aligned} \quad (2)$$

where Q_1^u and Q_2^u are the current-current operators, and the current-current operators Q_1^c and Q_2^c can be obtained from Q_1^u and Q_2^u through the substitution $u \to c$. $Q_3, \ldots, Q_6$ are the QCD penguin operators, whereas $Q_7, \ldots, Q_{10}$ are the electroweak penguin operators. The quark $q = s$ for $b \to s$ transition; $(V \pm A)$ refer to $\gamma_\mu(1 \pm \gamma_5)$.

The matrix elements are:

$$< \mathbf{Q}^T(\mu) \cdot \mathbf{C}(\mu) > \equiv < \mathbf{Q}^T >_0 \cdot \mathbf{C}'(\mu)\,, \quad (3)$$

where $< \mathbf{Q} >_0$ denotes the tree-level matrix elements of these operators, and $\mathbf{C}'(\mu)$ are defined as follows:

$$\begin{aligned}
&C_1' = \overline{C}_1, && C_2' = \overline{C}_2, && C_3' = \overline{C}_3 - P_s/3,\\
&C_4' = \overline{C}_4 + P_s, && C_5' = \overline{C}_5 - P_s/3, && C_6' = \overline{C}_6 + P_s,\\
&C_7' = \overline{C}_7 + P_e, && C_8' = \overline{C}_8, && C_9' = \overline{C}_9 + P_e,\\
&C_{10}' = \overline{C}_{10}.
\end{aligned} \quad (4)$$

Here $P_{s,e}$ are given by

$$\begin{aligned}
P_s &= \frac{\alpha_s}{8\pi}\overline{C}_2(\mu)\left[\tfrac{10}{9} - G(m_q,q,\mu)\right], \quad (5)\\
P_e &= \frac{\alpha_{em}}{9\pi}\left(3\overline{C}_1 + \overline{C}_2(\mu)\right)\left[\tfrac{10}{9} - G(m_q,q,\mu)\right],\\
G(m,q,\mu) &= -4\int_0^1 dx\ x(1-x)\ln\left[\frac{m^2 - x(1-x)q^2}{\mu^2}\right],
\end{aligned}$$

where q represents $u,\ c$, and $q^2 = m_b^2/2$. Numerical values of the scheme-independent renormalization Wilson Coefficients $\overline{C}_i(\mu)$ at $\mu = O(m_b)$ are [9]:

$$\begin{aligned}
&\bar{c}_1 = -0.313, && \bar{c}_2 = 1.150, && \bar{c}_3 = 0.017,\\
&\bar{c}_4 = -0.037, && \bar{c}_5 = 0.010, && \bar{c}_6 = -0.046,\\
&\bar{c}_7 = -0.001\cdot\alpha_{em}, && \bar{c}_8 = 0.049\cdot\alpha_{em},\\
&\bar{c}_9 = -1.321\cdot\alpha_{em}, && \bar{c}_{10} = 0.267\cdot\alpha_{em}.
\end{aligned} \quad (6)$$

(1) **Without final-state interaction**

In $B_u^- \to \pi^- \bar{K^0}$ decay, only penguin diagrams contribute. As is commonly agreed in the present literatures, the annihilation diagram contributions are neglected because of V_{ub} and the form factor suppression. In $\bar{B_d^0}(b\bar{d}) \to \pi^+ K^-$ decay, both tree and penguin diagrams contribute. The amplitudes of these two decays are:

$$\begin{aligned}
&A^{dir}(B_u^- \to \pi^- \bar{K^0})\\
&= \frac{G_F}{\sqrt{2}}\sum_{q=u,c} v_q\left[a_3 + \frac{2M_{\bar{K^0}}^2}{(m_s+m_d)(m_b-m_d)} \times\left(a_5 - \frac{1}{2}a_7\right) - \frac{1}{2}a_9\right]M^{\bar{K^0}\pi^-}
\end{aligned} \quad (7)$$

$$\begin{aligned}
&A^{dir}(\bar{B^0} \to \pi^+ K^-)\\
&= \frac{G_F}{\sqrt{2}}\sum_{q=u,c} v_q\left[a_1\delta_{uq} + a_3 + \frac{2M_{K^-}^2}{(m_s+m_u)(m_b-m_u)} \times(a_5 + a_7) + a_9\right]M^{K^-\pi^+},
\end{aligned}$$

where "dir" means direct decay without final-state interactions, and

$$\begin{aligned}
M^{\bar{K^0}\pi^-} &\equiv <\bar{K^0}|(\bar{s}d)_{V-A}|0><\pi^-|(\bar{d}b)_{V-A}|B_u^->\\
&= -if_K F_0^{B_u^-\pi}(M_{K^0}^2)(M_{B_u}^2 - M_{\pi^-}^2)\\
M^{K^-\pi^+} &\equiv <K^-|(\bar{s}u)_{V-A}|0><\pi^+|(\bar{u}b)_{V-A}|\bar{B^0}>\\
&= -if_K F_0^{B^0\pi}(M_{K^-}^2)(M_{B^0}^2 - M_{\pi^+}^2),
\end{aligned} \quad (8)$$

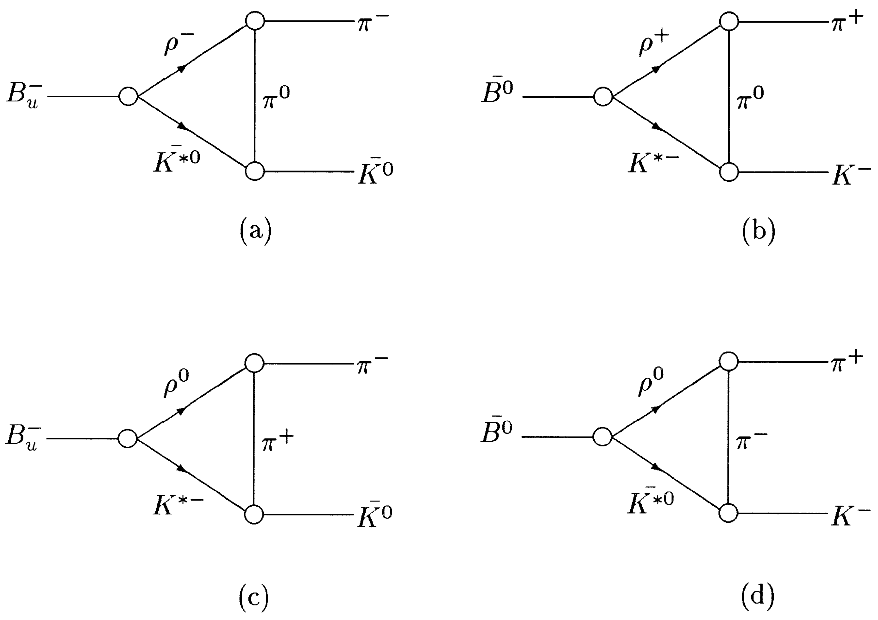

Fig. 1. Final-state interactions in $B \to \rho K^* \to \pi K$ due to single pion exchange; **a, b** the neutral pion exchange case; **c, d** the charged pion exchange case

Table 1. The amplitude and branching ratios of the direct $B \to \pi K$ decays, where "Tree" means the tree diagram contribution only; "penguin" means the penguin diagram contribution only. "Tree+penguin" means tree plus penguin diagram contributions

Decay Mode	N_c	$A^{dir}(10^{-9}GeV)$			Br	
		Tree	Penguin	Tree+Penguin	Tree	Tree+Penguin
$B_u^- \to \pi^- \bar{K^0}$	$N_c = \infty$	0	$6.53 + i36.1$	$6.53 + i36.1$	0	1.18×10^{-5}
$\bar{B^0} \to \pi^+ K^-$	$N_c = \infty$	-7.88+i2.78	$6.51 + i34.8$	$-1.36 + i37.6$	5.93×10^{-7}	1.20×10^{-5}
$B_u^- \to \pi^- \bar{K^0}$	$N_c = 3$	0	$5.78 + i31.2$	$5.78 + i31.2$	0	8.79×10^{-6}
$\bar{B^0} \to \pi^+ K^-$	$N_c = 3$	-7.16+i2.53	$5.82 + i32.0$	$-1.34 + i34.5$	4.9×10^{-7}	1.01×10^{-6}
$B_u^- \to \pi^- \bar{K^0}$	$N_c = 2$	0	$4.87 + i27.8$	$4.87 + i27.8$	0	6.94×10^{-6}
$\bar{B^0} \to \pi^+ K^-$	$N_c = 2$	-6.8+i2.4	$5.01 + i29.7$	$-1.79 + i32.1$	4.43×10^{-7}	8.81×10^{-6}

and

$$\begin{aligned} a_{2i-1} &= C'_{2i-1}/N_c + C'_{2i}\,, \\ a_{2i} &= C'_{2i-1} + C'_{2i}/Nc\,. \end{aligned} \quad (9)$$

Because $|v_c|/|v_u| >> 1$, the tree-diagram contributions to the decay amplitude are small compared to the penguin diagrams. The factorization approximation and BSW model [7] are used to evaluate the matrix elements in (8). Table 1 shows the calculation results in the standard method for the above two direct decays. The non-factorization effects have been considered with the choice $N_c = \infty, 3, 2$.

(2) **With final-state interaction**

In $B \to \rho K^* \to \pi K$ processes, the exchanged pions can be neutral or charged as is shown in Fig. 1. For charged-pion exchange the decay $B_u^{\mp} \to \pi^{\mp} K$ can get the tree diagram contribution in the intermediate state $B \to \rho K^* \to \pi K$.

We take the decay $B_u^- \to \rho^- \bar{K^{*0}} \to \pi^- \bar{K^0}$ as an example to show how to calculate the final-state interaction effects.

The amplitude for $B_u^- \to \rho^- \bar{K^{*0}}$ decay is the following:

$$A(B_u^- \to \rho^- \bar{K^{*0}}) = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} v_q (a_3 - \frac{1}{2} a_9) M^{\bar{K^{*0}}\rho^-}\,, \quad (10)$$

where

$$\begin{aligned} M^{\bar{K^{*0}}\rho^-} &\equiv < \bar{K^{*0}}|(\bar{s}d)_{V-A}|0 >< \rho^-|(\bar{d}b)_{V-A}|B_u^- > \\ &= \frac{2M_{K^{*0}}}{M_{B_u} + M_{\rho^-}} f_{K^*} V^{B_u\rho}(M_{K^{*0}}^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu}_{K^{*0}}\epsilon^{\nu}_{\rho^-} p^{\rho}_{K^{*0}} p^{\sigma}_{\rho^-} \\ &\quad + iM_{K^{*0}}(M_{B_u} + M_{\rho^-}) f_{K^*} A_1^{B_u\rho}(M_{K^{*0}}^2)(\epsilon_{K^{*0}} \cdot \epsilon_{\rho^-}) \\ &\quad - i\frac{2M_{K^{*0}}}{M_{B_u} + M_{\rho^-}} f_{K^*} A_2^{B_u\rho}(M_{K^{*0}}^2)(\epsilon_{K^{*0}} \cdot p_B)(\epsilon_{\rho^-} \cdot p_B). \end{aligned} \quad (11)$$

Following [6], the single pion exchange in the t-channel provides a significant contribution to FSI and would dominate. To get the absorptive part of the loop as is shown in Fig. 1a, the way to make cuts is: let ρ^- and $\bar{K}^{0*}$ be on-shell, and leave the exchanged pion to be off-shell.

In the B_u^- rest frame, where $p_{B_u} = (M_{B_u}, 0)$, the matrix element written in (11) is recast into the following

Table 2. The amplitue for the final-state interactions $B \to \rho K^* \to \pi K$

Decay Mode	N_c	$A^{FSI}(10^{-9}GeV)$			A^{FSI}/A^{dir}
		Tree	Penguin	Tree+Penguin	
$B_u^- \to \rho^- \bar{K^{*0}} \to \pi^- \bar{K^0}$	$N_c = \infty$	0	$9.86 - i1.86$	$9.86 - i1.86$	$-0.27e^{i89.6^0}$
$\bar{B^0} \to \rho^+ K^{*-} \to \pi^+ K^-$	$N_c = \infty$	$1.22 + i3.47$	$9.35 - i1.86$	$10.6 + i1.60$	$-0.28e^{i96.5^0}$
$B_u^- \to \rho^0 K^{*-} \to \pi^- \bar{K^0}$	$N_c = \infty$	$0.57 + i1.6$	$7.34 - i2.83$	$7.9 - i1.22$	$-0.22e^{i91.5^0}$
$\bar{B^0} \to \rho^0 \bar{K^{*0}} \to \pi^+ K^-$	$N_c = \infty$	$-0.25 - i0.72$	$-5.06 + i1.39$	$-5.32 + i0.67$	$0.14e^{i80.8^0}$
$B_u^- \to \rho^- \bar{K^{*0}} \to \pi^- \bar{K^0}$	$N_c = 3$	0	$8.2 - i1.65$	$8.2 - i1.65$	$-0.26e^{i89.1^0}$
$\bar{B^0} \to \rho^+ K^{*-} \to \pi^+ K^-$	$N_c = 3$	$1.11 + i3.15$	$8.61 - i1.67$	$9.72 + i1.49$	$-0.28e^{i96.5^0}$
$B_u^- \to \rho^0 K^{*-} \to \pi^- \bar{K^0}$	$N_c = 3$	$0.81 + i2.31$	$6.86 - i2.74$	$7.68 - i0.44$	$-0.24e^{i97.2^0}$
$\bar{B^0} \to \rho^0 \bar{K^{*0}} \to \pi^+ K^-$	$N_c = 3$	$0.06 + i0.16$	$-3.82 + i1.11$	$-3.77 + i1.27$	$0.12e^{i69.2^0}$
$B_u^- \to \rho^- \bar{K^{*0}} \to \pi^- \bar{K^0}$	$N_c = 2$	0	$7.1 - i1.38$	$7.1 - i1.38$	$-0.26e^{i89.0^0}$
$\bar{B^0} \to \rho^+ K^{*-} \to \pi^+ K^-$	$N_c = 2$	$1.06 + i2.99$	$8.01 - i1.43$	$9.06 + i1.56$	$-0.29e^{i96.6^0}$
$B_u^- \to \rho^0 K^{*-} \to \pi^- \bar{K^0}$	$N_c = 2$	$0.94 + i2.66$	$6.58 - i2.48$	$7.52 + i0.18$	$-0.27e^{i98.6^0}$
$\bar{B^0} \to \rho^0 \bar{K^{*0}} \to \pi^+ K^-$	$N_c = 2$	$0.21 + 0.60$	$-3.01 + i0.86$	$-2.8 + i1.46$	$0.10e^{i59.2^0}$

form for the process $B_u^- \to \rho^- \bar{K^{*0}} \to \pi^- \bar{K^0}$:

$$\begin{aligned} M_{FSI}^{\bar{K^0}\pi^-} &= \frac{1}{2}\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-p_B) \\ &\quad \times < \pi^- \bar{K^0}|S|\rho^- \bar{K^{*0}} > M^{\bar{K^{*0}}\rho^-} \\ &= \int \frac{|\vec{p}\,|}{16\pi M_{B_u}} d(cos\theta)\frac{iF(p_{\pi^0}^2)}{(p_{\pi^0}^2 - M_{\pi^0}^2)} \\ &\quad \times [iM_{K^{*0}}(M_{B_u}+M_{\rho^-})f_{K^*}A_1^{B_u\rho}(M_{K^{*0}}^2)\cdot H_1 \\ &\quad - i\frac{2M_{K^{*0}}}{M_{B_u}+M_{\rho^-}}M_{B_u}^2 f_{K^*}A_2^{B_u\rho}(M_{K^{*0}}^2)\cdot H_2], \end{aligned} \tag{12}$$

where S is the S-matrix of strong interaction, θ is the angle between $\vec{p_1}$and $\vec{p_3}$, and

$$\begin{aligned} H_1 &= -4g_{\rho\pi\pi}g_{K^*K\pi}[(p_3\cdot p_4) - \frac{(p_2\cdot p_3)(p_2\cdot p_4)}{M_2^2} \\ &\quad - \frac{(p_1\cdot p_3)(p_1\cdot p_4)}{M_1^2} + \frac{(p_1\cdot p_2)(p_2\cdot p_3)(p_2\cdot p_4)}{M_1^2 M_2^2}] \\ H_2 &= -4g_{\rho\pi\pi}g_{K^*K\pi}[(p_3^0 p_4^0) - \frac{(p_2^0 p_3^0)(p_2\cdot p_4)}{M_2^2} \\ &\quad - \frac{(p_1^0 p_4^0)(p_1\cdot p_3)}{M_1^2} + \frac{(p_1^0 p_2^0)(p_2\cdot p_3)(p_2\cdot p_4)}{M_1^2 M_2^2}]. \end{aligned} \tag{13}$$

We set $p_1 = p_{K^*}$, $p_2 = p_\rho$, $p_3 = p_K$, $p_4 = p_\pi$, $M_1 = M_{K^*}$, $M_2 = M_\rho$, $M_3 = M_K$, $M_4 = M_\pi$.

So, the amplitude of $B_u^- \to \rho^- \bar{K^{*0}} \to \pi^- \bar{K^0}$ is:

$$A^{FSI}(B_u^- \to \pi^- \bar{K^0}) = \frac{G_F}{\sqrt{2}}\sum_{q=u,c} v_q(a_3 - \tfrac{1}{2}a_9)M_{FSI}^{\bar{K^0}\pi^-}. \tag{14}$$

The factor $F(p_{\pi^0}^2)$ in (12) is an off-shell form factor for the vertices $K^*K\pi$ and $\rho\pi\pi$. We take $F(p_{\pi^0}^2) = (\frac{\Lambda^2 - m_{\pi^0}^2}{\Lambda^2 - p_{\pi^0}^2})$, as is done in [6], where $\Lambda = 1.2 - 2.0$ GeV.

3 Numerical results

The parameters such as meson decay constants, form factors and quark masses needed for our calculations are taken as follows.

Meson decay constant [11,10]:
$f_\pi = 0.13$ GeV, $f_K = 0.16$ GeV, $f_{\rho^0}^{u\bar{u}} = 0.156$ GeV, $f_{K^*} = 0.221$ GeV.
Form factor [10]:

$$\begin{aligned} &F_0^{B\pi}(0) = 0.333, \quad F_1^{B\pi}(0) = 0.333, \\ &V^{BK^*}(0) = 0.369, \; A_1^{BK^*}(0) = 0.328, \\ &A_2^{BK^*}(0) = 0.331, \; V^{B\rho}(0) = 0.329, \\ &A_1^{B\rho}(0) = 0.283, \quad A_2^{B\rho}(0) = 0.283. \end{aligned}$$

Effective strong coupling constants [6]:
$g_{K^*K\pi} = 5.8$, $g_{\rho\pi\pi} = 6.1$.
Λ in the off-shell form factor $F(p_{\pi^0}^2)$: $\Lambda = 1.5$ GeV.
Quark mass [11]:
$m_u = 0.005$ GeV, $m_d = 0.01$ GeV, $m_s = 0.2$ GeV, $m_c = 1.5$ GeV, $m_b = 4.5$ GeV.
The Wolfenstein CKM parameters [12]:
$\lambda = 0.22$, $A = 0.8$, $\eta = 0.34$, $\rho = -0.12$.

Due to non-factorization effects, it is hard to choose the value of N_c, so all three cases are taken into account: $N_c = \infty, 3, 2$.

Corresponding numerical results are presented in Table 1 and Table 2.

4 Constraints on γ and $\mathcal{A}_{cp}^{dir}$

(1) **Without final-state interaction**

For direct decays of $B \to \pi K$, the amplitudes are:

$$\begin{aligned} A^{dir}(B^+ \to \pi^+ K^0) &= A_{cs}^+ - A_{us}^+ e^{i\gamma} e^{i\delta_+}, \\ A^{dir}(B^- \to \pi^- \bar{K^0}) &= A_{cs}^+ - A_{us}^+ e^{-i\gamma} e^{i\delta_+}, \\ A^{dir}(B^0 \to \pi^- K^+) &= A_{cs}^0 - A_{us}^0 e^{i\gamma} e^{i\delta_0}, \\ A^{dir}(\bar{B^0} \to \pi^+ K^-) &= A_{cs}^0 - A_{us}^0 e^{-i\gamma} e^{i\delta_0}, \end{aligned} \quad (15)$$

where δ_0 and δ_+ are CP-conserving strong phases.

The ratio R is defined by:

$$\begin{aligned} R &\equiv \frac{BR(B^0 \to \pi^- K^+) + BR(\bar{B^0} \to \pi^+ K^-)}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- \bar{K^0})} \\ &= (\frac{A_{cs}^0}{A_{cs}^+})^2 \frac{1 - 2r_0 \cos\gamma \cos\delta_0 + r_0^2}{1 - 2r_+ \cos\gamma \cos\delta_+ + r_+^2}, \end{aligned} \quad (16)$$

where $r_0 = A_{us}^0/A_{cs}^0$, $r_+ = A_{us}^+/A_{cs}^+$. Neglecting the electro-weak penguin diagram contributions, we have $A_{us}^+ = 0$, i.e. $r_+ = 0$. According to SU(2) isospin symmetry of strong interaction, $A_{cs}^0/A_{cs}^+ \approx 1$. So,

$$R = 1 - 2r_0 \cos\gamma \cos\delta + r_0^2. \quad (17)$$

Following Fleischer and Mannel [3], we can obtain the inequality

$$R \geq \sin^2\gamma. \quad (18)$$

This is the Fleischer-Mannel relation.

Direct CP violation in $B^\pm \to \pi^\pm K$ is defined through the CP asymmetry:

$$\begin{aligned} \mathcal{A}_{\rm CP}^{\rm dir} &\equiv \frac{BR(B^+ \to \pi^+ K^0) - BR(B^- \to \pi^- \bar{K^0})}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- \bar{K^0})} \\ &= \frac{2r_+ \sin\gamma \sin\delta_+}{1 - 2r_+ \cos\gamma \cos\delta_+ + r_+^2}. \end{aligned} \quad (19)$$

where "dir" means direct CP asymmetry. In direct decays, it is small, as it follows from (19):

$$\mathcal{A}_{\rm CP}^{\rm dir} \leq \mathcal{O}(\lambda^2). \quad (20)$$

(2) **With final-state interaction**

When final-state interactions are considered, the amplitude of $B \to \pi K$ process is changed to:

$$\begin{aligned} &A(B^+ \to \pi^+ K^0) \\ &= A^{dir}(B^+ \to \pi^+ K^0) + A^{FSI}(B^+ \to \rho^+ K^{*0} \to \pi^+ K^0) \\ &\quad + A^{FSI}(B^+ \to \rho^0 K^{*+} \to \pi^+ K^0) \\ &= (A_{cs}^+ - A_{us}^+ e^{i\gamma} e^{i\delta_+})(1 + A_1 e^{i\delta_1} + A_3 e^{i\gamma} e^{i\delta_3}) \\ &A(B^0 \to \pi^- K^+) \\ &= A^{dir}(B^0 \to \pi^- K^+) + A^{FSI}(B^0 \to \rho^- K^{*+} \to \pi^- K^+) \\ &\quad + A^{FSI}(B^0 \to \rho^0 K^{*0} \to \pi^- K^+) \\ &= (A_{cs}^0 - A_{us}^0 e^{i\gamma} e^{i\delta_0})(1 + A_2 e^{i\delta_1}), \end{aligned} \quad (21)$$

where A_1, A_2, A_3 are final-state interaction amplitudes, and δ_1, δ_2, δ_3 are the strong phases caused by the final-state interactions, i.e. the phase shifts of the inelastic rescattering. The term $A_3 e^{i\gamma} e^{i\delta_3}$ is the contribution from the tree diagram describing the process $B^+ \to \rho^0 K^{*+} \to \pi^+ K^0$. Our numerical calculations give $A_1 = 0.5$, $A_2 = 0.15$, $A_3 = 0.05$, $\delta_1 \approx 90^0$, $\delta_2 \approx 90^0$. The strong phases are equal to 90^0, because we calculate only the absorptive part of the hadron loop caused by the final-state interactions. We will come back to this point in the last section. The value $A_1 = 0.5$ is greater than $A_2 = 0.15$, that is encouraging for our mechanism, because it can explain that the experimental branching ratio for $B^\pm \to \pi^\pm K$ is larger than for $B^0 \to \pi^\mp K^\pm$.

The ratio R in (16) is changed into

$$R = \frac{1 + A_2^2 + 2A_2 \cos\delta_2}{1 + A_1^2 + 2A_1 \cos\delta_1}(1 - 2r_0 \cos\gamma \cos\delta_0 + r_0^2). \quad (22)$$

If R' is defined as

$$R' = 1 - 2r_0 \cos\gamma \cos\delta_0 + r_0^2, \quad (23)$$

then it follows from (22), (23):

$$R' = \frac{1 + A_1^2 + 2A_1 \cos\delta_1}{1 + A_2^2 + 2A_2 \cos\delta_2} R. \quad (24)$$

So, the Fleischer-Mannel relation is changed to

$$R' \geq \sin^2\gamma. \quad (25)$$

At $A_1 = 0.5$, $A_2 = 0.15$, $\delta_1 = 90^0$, $\delta_2 = 90^0$ we get $R' = 1.25R$. The boundary relation (18) is modified in as much as 25%, when only the absorptive part of the hadron loop is under consideration. No doubt, the dispersive part of the loop will also contribute to the modification. Consider an example, when the dispersive part of the amplitude is equal to the absorptive one, $A_1 = 0.5\sqrt{2}$, $A_2 = 0.15\sqrt{2}$, $\delta_1 = 45^0$, $\delta_2 = 45^0$, then $R' = 1.58R$. So, the boundary relation is modified significantly, in about 58%.

For direct CP asymmetry,

$$\mathcal{A}_{\rm CP}^{\rm dir} \approx 2A_3 \sin\gamma \sin\delta_+. \quad (26)$$

The final-state interaction can provide about $5\% - 10\%$ direct CP asymmetry. But since the relative sign cannot be fixed by the theory, we are unable to determine whether the correction is constructive or destructive.

5 Conclusion and discussion

Numerical results presented in Table 1 and Table 2 show that final-state interactions due to the single pion exchange in $B \to \rho K^* \to \pi K$ processes comprise $10\% - 30\%$ in respect to the direct decay amplitude. This result is based only on the consideration of the absorptive part of the hadron loop caused by the final-state interactions. The dispersive part of the loop is difficult to calculate because of the ultraviolet divergence. The elastic and inelastic rescattering caused by vector trajectory(ρ, ω, and K^*)

exchange can give additional contributions, and there are many multiparticle intermediate states which cannot be neglected [4]. Because of these uncertainties, the simple Fleischer-Mannel relation $R \geq \sin^2 \gamma$ is strongly modified by the final-state interactions. We think it is difficult to get reliable information on the weak angle γ in $B \to \pi K$ processes. The $5\% - 10\%$ direct CP asymmetry can be generated by final-state interactions. Moreover, as is discussed above, we only consider the absorptive part of the hadron loop in this work; there are still many uncertainties of theoretical predictions on the constraint of γ and A_{CP}^{dir}. At present stage, there is no reliable renormalization scheme for obtaining correct dispersive part of the hadron loop, and it is a well-known fact for evaluating the loops in the chiral Lagrangian theories. We will try some phenomenological ways to carry out the renormalization elsewhere[13].

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